CN510 Assignment 1: The Leaky Integrator

Due: September 24, 2007

The goal of this assignment is to familiarize you with numerical integration and refresh your skill at analyzing ordinary differential equations (ODEs). You are asked to simulate and analyze a leaky integrator, a simple model of short-term memory (STM).

The leaky integrator model of neural activity is so called because the neuron is assumed to integrate its input over time while decaying, or leaking, at a rate proportional to its activity. The leaky integrator is represented by the following equation:

$$dx/dt = -Ax + I \qquad (1)$$

where I is an input stimulus and $x = x(t)$ is the STM activity of the neuron. The constant A represents the spontaneous decay rate of the neuron.

(a) Let $A = 1$, and suppose $x(0) = 0$, so that the neuron initially has no activity. Suppose an input of $I = 5$ is presented from time $t = 1$ to time $t = 6$, but at all other times, $I = 0$. Numerically integrate equation (1) from time $t = 0$ to $t = 10$. Plot the resulting curve. What is its asymptote while input is present?

(b) Set $A = 2$ and repeat part (a). What has changed?

(c) Solve equation (1) for the equilibrium value of x assuming that a constant input of size I is present. Do this by setting the left-hand side to zero and solving for x. Does the result agree with your simulations?

(d) Assume I is constant and initial conditions $x(0) = 0$, so that equation (1) is a first-order constant coefficient linear ODE. Analytically solve equation (1), to obtain an explicit equation for $x(t)$ showing the dependence of x on A and I.

To do the numerical integration in parts (a) and (b), Euler's method suffices. (That is, to approximate $x(t)$ you may use the algorithm:

$$x(t + \Delta t) = x(t) + (dx/dt) \cdot \Delta t \qquad (2)$$

where Δt is a small time step.) Your report should include the source code you used to do the integration, two plots (one for part (a) and one for part (b)), and the solutions to parts (c) and (d). Be sure also to answer all questions.